

High School Exit Examination Spring 2000 Field Test Content

**California Mathematics
Content Standards, Grades 6 – Algebra I**

**Proposed by the High School
Exit Examination Standards Panel**

Note

Please note that the spring 2000 HSEE field test will assist the Superintendent and the State Board of Education (SBE) in determining the content of the test that the SBE is required to adopt by October 1, 2000. There will be a second field test in the fall. The content of the HSEE that will be administered to grade 9 students on a voluntary basis in spring 2001 will be determined from these two field tests.

Revised March 2000

Grade 6—Statistics, Data Analysis, and Probability

1.0 Students compute and analyze statistical measurements for data sets:

1.1 Compute the range, mean, median, and mode of data sets.

1.2 Understand how additional data added to data sets may affect these computations of measures of central tendency.

1.3 Understand how the inclusion or exclusion of outliers affects measures of central tendency.

1.4 Know why a specific measure of central tendency (mean, median, mode) provides the most useful information in a given context.

2.0 Students use data samples of a population and describe the characteristics and limitations of the samples:

2.1 Compare different samples of a population with the data from the entire population and identify a situation in which it makes sense to use a sample.

2.2 Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling) and which method makes a sample more representative for a population.

2.3 Analyze data displays and explain why the way in which the question was asked might have influenced the results obtained and why the way in which the results were displayed might have influenced the conclusions reached.

2.4 Identify data that represent sampling errors and explain why the sample (and the display) might be biased.

2.5 Identify claims based on statistical data and, in simple cases, evaluate the validity of the claims.

3.0 Students determine theoretical and experimental probabilities and use these to make predictions about events:

3.1 Represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome.

3.2 Use data to estimate the probability of future events (e.g., batting averages or number of accidents per mile driven).

3.3 Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable; know that if P is the probability of an event, $1-P$ is the probability of an event not occurring.

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Grade 6—Statistics, Data Analysis, and Probability (continued)

3.4 Understand that the probability of either of two disjoint events occurring is the sum of the two individual probabilities and that the probability of one event following another, in independent trials, is the product of the two probabilities.

3.5 Understand the difference between independent and dependent events.

Grade 7—Number Sense

1.0 Students know the properties of, and compute with, rational numbers expressed in a variety of forms:

1.1 Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10) with approximate numbers using scientific notation.

1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers

1.3 Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.

1.4 Differentiate between rational and irrational numbers.

1.5 Know that every rational number is either a terminating or repeating decimal and be able to convert terminating decimals into reduced fractions.

1.6 Calculate the percentage of increases and decreases of a quantity.

1.7 Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.

2.0 Students use exponents, powers, and roots and use exponents in working with fractions:

2.1 Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base.

2.2 Add and subtract fractions by using factoring to find common denominators.

2.3 Multiply, divide, and simplify rational numbers by using exponent rules.

2.4 Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.

2.5 Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.

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Grade 7—Algebra and Functions	
1.0	Students express quantitative relationships by using algebraic terminology, expressions, equations, inequalities, and graphs:
1.1	Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A)
1.2	Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$.
1.3	Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.
1.4	Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.
1.5	Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.
2.0	Students interpret and evaluate expressions involving integer powers and simple roots:
2.1	Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.
2.2	Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.
3.0	Students graph and interpret linear and some nonlinear functions:
3.1	Graph functions of the form $Y=nx^2$ and $y=nx^3$ and use in solving problems.
3.2	Plot the values from the volumes of three-dimensional shapes for various values of the edge lengths (e.g., cubes with varying edge lengths or a triangle prism with a fixed height and a equilateral triangle base of varying lengths).
3.3	Graph linear functions, noting that the vertical change (change in y-value) per unit of horizontal change (change in x-value) is always the same and know that the ratio (“rise over run”) is called the slope of a graph.
3.4	Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of a line equals the quantities.

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Grade 7—Algebra and Functions (continued)

4.0 Students solve simple linear equations and inequalities over the rational numbers:

4.1 Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.

4.2 Solve multistep problems involving rate, average speed, distance, and time or a direct variation.

Grade 7—Measurement and Geometry

1.0 Students choose appropriate units of measure and use ratios to convert within and between measurement systems to solve problems

1.1 Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).

1.2 Construct and read drawings and models made to scale.

1.3 Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.

2.0 Students compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects. They know how perimeter, area and volume are affected by changes of scale:

2.1 Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms and cylinders.

2.2 Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.

2.3 Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and volume is multiplied by the cube of the scale factor.

2.4 Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot=144 square inches or $[1 \text{ ft}^2]=[144 \text{ in}^2]$, 1 cubic inch is approximately 16.38 cubic centimeters or $[1 \text{ in}^3]=[16.38 \text{ cm}^3]$)

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Grade 7—Measurement and Geometry (continued)

- 3.0 Students know the Pythagorean theorem and deepen their understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures:
- 3.1 Identify and construct basic elements of geometric figures (e.g., altitudes, mid-points, diagonals, angle bisector, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.
 - 3.2 **Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.**
 - 3.3 **Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.**
 - 3.4 **Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.**
 - 3.5 Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.
 - 3.6 Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).

Grade 7—Statistics, Data Analysis, and Probability

- 1.0 Students collect, organize, and represent data sets that have one or more variables and identify relationships among variables within a data set by hand and through the use of an electronic spreadsheet software program:
- 1.1 **Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.**
 - 1.2 **Represent two numerical variables on a scatterplot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).**
 - 1.3 **Understand the meaning of, and be able to compute the minimum, the lower quartile, the median, the upper quartile, and the maximum of a data set.**

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Grade 7—Mathematical Reasoning	
1.0	Students make decisions about how to approach problems:
1.1	Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.
1.2	Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.
1.3	Determine when and how to break a problem into simpler parts.
2.0	Student use strategies, skills, and concepts in finding solutions:
2.1	Use estimation to verify the reasonableness of calculated results.
2.2	Apply strategies and results from simpler problems to more complex problems.
2.3	Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.
2.4	Make and test conjectures by using both inductive and deductive reasoning.
2.5	Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models to explain mathematical reasoning.
2.6	Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.
2.7	Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.
2.8	Make precise calculations and check the validity of the results from the context of the problem.
3.0	Students determine a solution is complete and move beyond a particular problem by generalizing to other situations:
3.1	Evaluate the reasonableness of the solution in the context of the original situation.
3.2	Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.
3.3	Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.

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Algebra I

1.0	Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable:
1.1	Students use properties of numbers to demonstrate whether assertions are true or false.
2.0	Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.
3.0	Students solve equations and inequalities involving absolute values.
4.0	Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x-5) + 4(x-2) = 12$.
5.0	Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.
6.0	Students graph a linear equation and compute the x- and y-intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by $2x + 6y < 4$).
7.0	Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations by using the point-slope formula.
8.0	Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.
9.0	Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.
10.0	Students add, subtract, multiply, and divide monomials and polynomials. Students solve multi-step problems, including word problems, by using these techniques.
11.0	Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.
12.0	Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.
13.0	Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.
14.0	Students solve a quadratic equation by factoring or completing the square.

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Algebra I (continued)
15.0 Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.
16.0 Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.
17.0 Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.
18.0 Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.
19.0 Students know the quadratic formula and are familiar with its proof by completing the square.
20.0 Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.
21.0 Students graph quadratic functions and know that their roots are the x-intercepts.
22.0 Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points.
23.0 Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.
24.0 Students use and know simple aspects of a logical argument:
24.1 Students explain the difference between inductive and deductive reasoning and identify and provide examples of each.
24.2 Students identify the hypothesis and conclusion in logical deduction.
24.3 Students use counter examples to show that an assertion is false and recognize that a single counter example is sufficient to refute an assertion.
25.0 Students use properties of the number system to judge the validity of results, to justify each step of a procedure, and to prove or disprove statements:
25.1 Students use properties of numbers to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions.
25.2 Students judge the validity of an argument according to whether the properties of the real number system and the order of operations have been applied correctly at each step.
25.3 Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or equations or inequalities, students determine whether the statement is true sometimes, always, or never.

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